CONFIDENTIAL

UNIVERSITY COLLEGE TATI (UC TATI)

FINAL EXAMINATION QUESTION BOOKLET		
COURSE CODE	:	BGE 1133
COURSE	:	CALCULUS
SEMESTER/SESSION	:	1 – 2023/2024
DURATION	:	3 HOURS

Instructions:

1. This booklet contains **FIVE (5)** questions in SECTION A, **THREE (3)** questions in SECTION B and **TWO (2)** questions in SECTION C. Answer **ALL** questions.
2. All answers should be written in answer booklet.
3. Write legibly and draw sketches wherever required.
4. If in doubt, raise your hands and ask the invigilator.

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO

THIS BOOKLET CONTAINS 7 PRINTED PAGES INCLUDING COVER PAGE

SECTION A (50 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Differentiate each of the following function with respect to x.

a) $y = 7x^2 - 6e^{4x} + 3x$ (2 marks)

b) $y = \frac{4}{x^5} + 8x^{1/3} - \ln 4x$ (2 marks)

c) $y = 5\sin(3x) - \tan(x) + 10$ (2 marks)

d) $y = x^3 e^{7x}$ (use Product Rule) (3 marks)

e) $y = \frac{\cos(4x)}{5x^3}$ (use Quotient Rule) (3 marks)

QUESTION 2

Integrate each of the following functions.

a) $\int \left(e^{-2x} + 2x^5 - 3\right) dx$ (2 marks)

b) $\int \left(\sin(x) + 5\cos(10x)\right) dx$ (2 marks)

c) $\int \left(\frac{x^5 - x^3 - 3}{x^2}\right) dx$ (3 marks)

d) $\int 6x\left(1 - 4x^2\right)^4 dx$ (use By Substitution method) (4 marks)

e) $\int 5x^2 \ln(x)\, dx$ (use By Parts method) (4 marks)

QUESTION 3

a) If $x=(3t-1)^3$ and $y=5(t-1)^{-3}$, find $\frac{dy}{dx}$ in terms of t. (5 marks)

b) Find $\frac{dy}{dx}$ implicitly for $3x+2x^2y^2=1$. (5 marks)

QUESTION 4

a) Determine whether or not the equation is separable.

$$\frac{dy}{dx}-3e^{x-y}=0$$ (3 marks)

b) Show that the equation is exact.

$$\left(e^{4x}+2xy^2\right)dx+\left(\cos y+2x^2y\right)dy=0$$ (3 marks)

c) Find a general solution of the following second order homogeneous linear differential equation (ODE).

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}=-2y$$ (4 marks)

QUESTION 5

Find the inverse Laplace transforms for the following function.

$$L^{-1}\left\{\frac{4}{s}+\frac{s}{s^2+9}-\frac{3}{s+5}\right\}$$ (3 marks)

SECTION B (30 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Solve $\int \frac{x^2 - x + 2}{(x+1)(x^2+9)}\,dx.$ (8 marks)

Hint: $\int \frac{1}{(x^2 + a^2)}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

QUESTION 2

a) Solve $2\frac{dy}{dx} = \frac{y(x+1)}{x}$ using separation of the variables. (5 marks)

b) Find a general solution of the differential equation, $y' + \frac{y}{x} = \frac{e^x}{x}$ using the integrating factor method. (5 marks)

QUESTION 3

Consider the following second order linear differential equation (ODE).

$$2y'' + 7y' + 3y = 20\sin 2x$$

a) Find the complementary function, y_c. (3 marks)

b) Find the particular function, y_p. (8 marks)

c) Find the general solution of the given differential equation above. (1 mark)

SECTION C (20 MARKS)

INSTRUCTION: ANSWER ALL QUESTIONS.

QUESTION 1

Find the equations of tangent and normal lines of $x^2 + x^3 + 3xy + y^2 = 5$ at the point $(1,1)$. (9 marks)

QUESTION 2

Use Laplace transforms to solve the following differential equation.

$$y'' + 4y' = e^{-2t}, \qquad y(0) = 0, \quad y'(0) = 6$$ (11 marks)

-------------------------------------- END OF QUESTION --------------------------------------

FORMULA

$\frac{d}{dx}(c)=0$	$m_T.m_N=-1$
$\frac{d}{dx}(x^n)=nx^{n-1}$	$y-y_1=m(x-x_1)$
$\frac{d}{dx}(kf(x))=k\frac{d}{dx}f(x)$	$\int dx=x+C$
$\frac{d}{dx}[f(x)]^n=n[f(x)]^{n-1}\cdot f'(x)$	$\int k\,f(x)\,dx=k\int f(x)dx+C$
$\frac{d}{dx}(\sin x)=\cos x$	$\int x^n dx=\frac{x^{n+1}}{n+1}+C, \quad n\neq -1$
$\frac{d}{dx}(\cos x)=-\sin x$	$\int (ax+b)^n dx=\frac{(ax+b)^{n+1}}{(n+1)a}+C, \quad n\neq -1$
$\frac{d}{dx}(\tan x)=\sec^2 x$	$\int \cos x\,dx=\sin x+C$
$\frac{d}{dx}(\sin f(x))=f'(x)\cos f(x)$	$\int \sin x\,dx=-\cos x+C$
$\frac{d}{dx}(\cos f(x))=-f'(x)\sin f(x)$	$\int \sec^2 x\,dx=\tan x+C$
$\frac{d}{dx}(\tan f(x))=f'(x)\sec^2 f(x)$	$\int \cos(ax+b)dx=\frac{\sin(ax+b)}{a}+C$
$\frac{d}{dx}(e^x)=e^x$	$\int \sin(ax+b)\,dx=-\frac{\cos(ax+b)}{a}+C$
$\frac{d}{dx}(e^{f(x)})=f'(x)e^{f(x)}$	$\int \sec^2(ax+b)\,dx=\frac{\tan(ax+b)}{a}+C$
$\frac{d}{dx}(\ln x)=\frac{1}{x}$	$\int e^x\,dx=e^x+C$
$\frac{d}{dx}(\ln f(x))=\frac{1}{f(x)}f'(x)$	$\int e^{ax+b}dx=\frac{1}{a}e^{ax+b}+C$
$\frac{d}{dx}(uv)=uv'+vu'$	$\int \frac{1}{x}\,dx=\ln\|x\|+C$
$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$	$\int \frac{1}{ax+b}dx=\frac{1}{a}\ln\|ax+b\|+C$
$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$	$\int u\,dv=uv-\int v\,du$
$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$	$\int \frac{1}{a^2+x^2}dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

$\int \frac{1}{a^2-x^2}dx = \frac{1}{a}\sin^{-1}\left(\frac{x}{a}\right)+C$	$ye^{\int p(x)dx} = \int q(x)e^{\int p(x)dx}\,dx$
$y = k_1 e^{m_1 x} + k_2 e^{m_2 x}$	$y = k_1 e^{mx} + k_2 x e^{mx}$
$y = e^{ax}\left(k_1 \sin bx + k_2 \cos bx\right)$	

TABLE OF PARTICULAR FUNCTION, y_p

Types of $g(x)$	Example of $g(x)$	y_p
Polynomial	$a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$	$A_n x^n + A_{n-1}x^{n-1} + \ldots + A_0$
Exponential	$e^{\alpha x}$	$Ae^{\alpha x}$
Trigonometric	$a\sin\beta x$ $a\cos\beta x$ $a\sin\beta x + a\cos\beta x$	$A\cos\beta x + B\sin\beta x$
Combination of types i,ii,iii	$e^{\alpha x} + x^2$ $e^{\alpha x} + a\sin\beta x$	$Ae^{\alpha x} + Bx^2 + Cx + D$ $Ae^{\alpha x} + B\sin\beta x + C\cos\beta x$

TABLE OF LAPLACE TRANSFORMS

No.	$f(t)$	$L\{f(t)\} = F(s), \quad s>0$
1.	a	$\frac{a}{s}$
2.	t	$\frac{1}{s^2}$
3.	$t^n, \ n = 2,3,\ldots$	$\frac{n!}{s^{n+1}}$
4.	e^{at}	$\frac{1}{s-a}$
5.	e^{-at}	$\frac{1}{s+a}$
6.	$\sin at$	$\frac{a}{s^2+a^2}$
7.	$\cos at$	$\frac{s}{s^2+a^2}$
8.	y'	$sL(y) - y(0)$
9.	y''	$s^2L(y) - sy(0) - y'(0)$